RESEARCH ARTICLE

Strongly B* - Continuous Multiset Functions In Multiset - Topological Spaces

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ABSTRACT: In this paper, we introduce the new concept of Sb^*- Open, closed and continuous multiset function in multiset topological space. Also study some interesting properties of Sb^*-closed and Sb^* −continuous mutiset function in multiset topological space

KEYWORDS: Multiset function, strongly b*- continuous M-set functions, strongly b*-openmaps and closed maps.

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1. INTRODUCTION

Levine [12] introduced the concept of generalized closed sets in topological spaces and a class of topological spaces called $T_{\frac{1}{2}}$ - spaces. Dunham [7] and Dunham and Levine [8] further studied some properties of generalized closed sets and $T_{\frac{1}{2}}$ - spaces. Strong forms of continuous maps have been introduced and investigated by several mathematicians. Strongly continuous maps, perfectly continuous maps, completely continuous maps, open continuous maps were introduced by Levine [14], Noiri [19], Munshi and Bassan [16] and Reilly and Vamanamurthy [21] respectively.

Semi continuous functions have been studied by several authors. Dontchev [5], Ganster and Reilly [6] introduced contra - continuous functions and regular set - connected functions. Erdal Ekici [9] introduced and studied a new class of functions called almost contra – pre - continuous functions which generalize classes of regular set – connected [6], contra – pre continuous [12], contra continuous [5], almost s - continuous [18] and perfectly continuous functions [19]. Sunil Jacob John and Girish introduced Multiset Topology [26]. In this paper, we introduce the new concept of Sb^*- Open, closed and continuous mutiset function in multiset topological space. Also study some interesting properties of Sb^*-closed and sb^* −continuous mutiset function in multiset topological space.

2. PRELIMINARIES

In this section the necessary basic definitions are studied.

Definition 2.1[21]: A map f: X → Y from a topological space X into a topological space Y is called semi- generalized continuous (sg-continuous) if f^{-1}(V) is sg- closed in X for every closed set V of Y.

Definition 2.2[2]: A function f: X → Y is said to be generalized continuous (g-continuous) if f^{-1} (V) is g-open in X for each open set V of Y.

Definition: 2.3[4]: A function f: X→ Y is said to be αg - continuous if f^{-1}(V) is αg - open in X for each open set V of Y.

Definition 2.4[20]: A subset A of a topological space (X, τ) is called a strongly b^*- closed set (briefly sb^*- closed) if cl(int(A)) ⊆ U whenever A ⊆ U and U is b open in X.

Definition 2.5[19]: Let X and Y be topological spaces. A map f: X → Y is called strongly b*-continuous (sb*- continuous). If the inverse image of every open set in Y is sb*- open in X.

Definition 2.6: Let X and Y be topological spaces. A map f: X → Y is called strongly b*- closed (sb*-closed) map, if the image of every closed set in X is sb*- closed set in Y.

Definition 2.7: A subset (X, τ) is said to be α-closed set, if cl(int(cl(A))) ⊆ A.

Definition 2.8: A subset R of $M \times M$ is said to be an M-set relation on M, if for every member $\left(\frac{m}{x}, \frac{n}{y}\right)$ of R has a count, product of $c_1(x, y)$and $c_2(x, y)$. we denote $\frac{m}{x}$ related to $\frac{n}{y}$ by $\frac{m}{x}$ R $\frac{n}{y}$.

Definition 2.9: An M-set relation f is called an M-

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set function, if for every element $\frac{m}{x}$ in Dom f, there is exactly one $\frac{n}{y}$ in Ran f such that $\left(\frac{m}{x}, \frac{n}{y}\right)$ is in f with the pair accruing as the product of $c_1(x, y)$and $c_2(x, y)$.

Definition 2.9: Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then τ is called a multiset topology of M if τ satisfies the following properties

(a) The M-set M and the empty M-set ϕ are in τ.
(b) The M-set union of the elements of any sub collections of τ is in τ.
(c) The M-set intersection of the elements of any finite sub collections of τ is in τ.

1. STRONGLY b*- CONTINOUS M-SET FUNCTIONS

Throughout this section X denote a non-empty set, $M \in [X]^w$and $C_M: X \to W$ where W is the set of all whole numbers.

Definition 3.1: Let (M, τ) be an topological space. A sub M-set $A \subseteq M$ is said to be a Sg −closed (semi-generalized) M-set if $\mathrm{Scl(A)} \subseteq \mathrm{U}$ whenever $A \subseteq U$ and U is an semi open M-set in (M, τ) with $C_{\mathrm{Scl(A)}}(\mathrm{x}) \leq C_{\mathrm{U}}(\mathrm{x})$ whenever $C_A(\mathrm{x}) \leq C_{\mathrm{U}}(\mathrm{x})$ for all $x \in X$. The complement of a sg −closed M-set is said to be a sg −open M-set.

Definition 3.2: Let (M, τ) be an topological space. A sub M-set $A \subseteq M$ is said to be a α −open M-set if $A \subseteq \mathrm{int(cl(int(A)))}$ with $C_A(\mathrm{x}) \leq C_{\mathrm{int}\left(\mathrm{cl(int(A))}\right)}(\mathrm{x})$ for all $x \in X$. The complement of a α −open M-set is said to be a α −closed M-set.

Example 3.3: Let X= {a, b} W = 1 and $M = \left\{\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right\}, \tau = \left\{\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}\right\}$. clearly τ is an M-topological space. Here the α −open M-sets are $\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}$ and the sg-open M-sets are $\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}$.

Definition 3.4: Let (M, τ) be an M-topological space. A sub M-set $A \subseteq M$ is said to be a αg −closed M-set if $\alpha\mathrm{cl(A)} \subseteq \mathrm{U}$ whenever $A \subseteq U$ and U is an open M-set in (M, τ) with $C_{\alpha(\mathrm{cl(A)})}(\mathrm{x}) \leq C_U(x)$ whenever $C_A(\mathrm{x}) \leq C_{\mathrm{U}}(\mathrm{x})$ for all $x \in X$. The complement of a αg −closed M-set is said to be a αg −open M-set.

Definition 3.5: Let (M, τ) be an M-topological space. A sub M-set $A \subseteq M$ is said to be a b −open M-set if $A \subseteq \mathrm{cl}\left(\mathrm{int(A)}\right) \cup \mathrm{int}\left(\mathrm{cl(A)}\right)$ with $C_A(\mathrm{x}) \leq C_{\mathrm{cl(int(A))} \cup \mathrm{int(cl(A))}}(\mathrm{x})$ for all $x \in X$. The complement of a b −open M-set is said to be b −closed M-set.

Definition 3.6: Let (M, τ) be an M-topological space. A sub M-set $A \subseteq M$ is said to be strongly b^* −closed (briefly sb^*- closed) M-set if $\mathrm{cl(int(A))} \subseteq \mathrm{U}$ whenever $A \subseteq U$ and U is b-open M-set in (M, τ) with $C_{\mathrm{cl(int(A))}}(\mathrm{x}) \leq C_{\mathrm{U}}$ whenever $C_A(x) \leq C_U(x)$for all $x \in X$. The complement of a sb^* −cloesd M-set is said to be a sb^* −open M-set.

Definition 3.7: Let (M, τ) be an M-topological space. Then the α closure of an M-set A is denoted by $\alpha cl(A)$ and defined as $\alpha cl(A)$ =∩{B: B ⊇ A for each $B \subseteq M$ is a $\alpha -$ closed $\mathrm{M-set}$} with $C_{\alpha cl(A)}(x)$ = min {B:B ⊇ A each $B \subseteq M$ is a $\alpha -$ closed $\mathrm{M-set}$} for all $x \in X$.

Example 3.8: Let X= {x, y}, W = 2 and $M = \left\{\frac{2}{x}, \frac{1}{y}\right\}, \tau = \left\{\mathrm{M}, \phi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$ clearly τ is an M-topology and the ordered pair (M, τ) is an M-topological space. Now the α −closed M-sets are $\mathrm{M}, \phi, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}\right\}$. Let $A = \left\{\frac{2}{x}\right\}$ be a sub M-set of M. Then $\alpha cl(A) = \left\{\frac{2}{x}\right\}$.

Example 3.9: Let X= {a, b, c}, W = 1 and $M = \left\{\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right\}, \tau = \left\{\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}\right\}$ clearly τ is an M-topology and the ordered pair (M, τ) is an M-topological space. Now the α −open M-sets are M, ϕ, $\left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}$ and the sg −open M-sets are $\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}$,

$\left\{\frac{1}{a}, \frac{1}{c}\right\}$ and the αg −open M-sets are $\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{c}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}$ and the b−open M-sets are $\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}$and sb^* −open M-sets are $\mathrm{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}$.

Definition 3.10: Let (M, τ) and (N, σ) be any two M topological spaces. Any M-set function $f: (M, \tau) \to (N, \sigma)$ is called generalized continuous (g-continuous) M-set function, if $f^{-1}(V)$ is g-open (resp g-closed) M-set in (M, τ) for each open (resp. closed) M-set V in (N, σ).

Example 3.11: Let X= {x, y, z}, W_1 = 2 and $Y = \{a, b\}$, $W_2 = 2$. Let $M = \left\{\frac{2}{x}, \frac{1}{y}, \frac{1}{z}\right\}$ and $N = \left\{\left\{\frac{2}{a}\right\}, \left\{\frac{1}{b}\right\}\right\}$be two M-sets. Let $\tau = \left\{\mathrm{M}, \phi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}\right\}$ and $\sigma = \left\{\mathrm{N}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{2}{a}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M, τ) and (N, σ) be two topological spaces. Now the g-open M-set of (M, τ) are $\mathrm{M}, \phi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$and the open M-set of (N, σ) are $\mathrm{N}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{2}{a}\right\}$. Let the M-set function $f: (M, \tau) \to (N, \sigma)$ be defined as $f =$

$\left\{\frac{\left\{\frac{2}{x},\frac{2}{a}\right\}}{4}, \frac{\left\{\frac{1}{y},\frac{2}{a}\right\}}{2}, \frac{\left\{\frac{1}{z},\frac{1}{b}\right\}}{1}\right\}$. Hence f is g-continuous M-set function, as each open M-set V in (N,σ) and $f^{-1}(V)$ is g-open M-set in (M,τ).

Definition 3.12: Let (M,τ) and (N,σ) be any two M topological spaces. Any M-set function $f:(M,\tau)\to(N,\sigma)$ is called αg −continuous M-set function, if $f^{-1}(V)$ is αg-open (resp αg-closed) M-set in (M,τ) for each open (resp. closed) M-set V in (N,σ).

Example 3.13: Let X= {a, b, c}, W_1 = 1 and $Y = \{x,y\}$, $W_2 = 2$. Let $M = \left\{\frac{1}{a},\frac{1}{b},\frac{1}{c}\right\}$ and $N = \left\{\left\{\frac{1}{x}\right\},\left\{\frac{2}{y}\right\}\right\}$ be two M-sets. Let $\tau = \left\{\mathrm{M},\phi,\left\{\frac{1}{a}\right\},\left\{\frac{1}{a},\frac{1}{c}\right\}\right\}$ and $\sigma = \left\{\mathrm{N},\phi,\left\{\frac{2}{y}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M,τ) and (N,σ) be two topological spaces. Now the αg-open M-set of (M,τ) are M, ϕ, $\left\{\frac{1}{a}\right\},\left\{\frac{1}{c}\right\},\left\{\frac{1}{a},\frac{1}{c}\right\}$. Let the M-set function $f:(M,\tau)\to(N,\sigma)$ be defined as $f = \left\{\frac{\left\{\frac{1}{a},\frac{1}{x}\right\}}{1}, \frac{\left\{\frac{1}{b},\frac{1}{x}\right\}}{1}, \frac{\left\{\frac{1}{c},\frac{2}{y}\right\}}{2}\right\}$. Hence f is αg-continuous M-set function, as the inverse image of every open M-set V in (N,σ) and $f^{-1}(V)$ is αg-open M-set in (M,τ).

Definition 3.14: Let (M,τ) and (N,σ) be any two M topological spaces. Any M-set function $f:(M,\tau)\to(N,\sigma)$ is called strongly b^* −continuous (sb^*- continuous) M-set function, if the inverse image of every open (resp. closed) M-set in (N,σ) is sb^* −open (resp sb^*-closed) M-set in (M,τ).

Example 3.15: Let X= {a, b}, W_1 = 2 and $Y = \{x,y\}$, $W_2 = 2$. Let $M = \left\{\frac{2}{a},\frac{1}{b}\right\}$ and $N = \left\{\left\{\frac{2}{x}\right\},\left\{\frac{1}{y}\right\}\right\}$ be two M-set.

Let $\tau = \left\{\mathrm{M},\phi,\left\{\frac{1}{a}\right\},\left\{\frac{1}{a},\frac{1}{b}\right\}\right\}$ and $\sigma = \left\{\mathrm{N},\phi,\left\{\frac{2}{x}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M,τ) and (N,σ) be two M-topological spaces. Now the sb^*-open M-set of (M,τ) are M, ϕ, $\left\{\frac{2}{a}\right\},\left\{\frac{1}{a}\right\},\left\{\frac{1}{b}\right\},\left\{\frac{1}{a},\frac{1}{b}\right\}$.

Let the M-set function $f:(M,\tau)\to(N,\sigma)$ be defined as $f = \left\{\frac{\left\{\frac{2}{a},\frac{2}{x}\right\}}{4}, \frac{\left\{\frac{1}{b},\frac{1}{y}\right\}}{1}\right\}$.

Hence f is sb^*-continuous M-set function, as the inverse image of every open M-set in (N,σ) is sb^*-open M-set in (M,τ).

Theorem 3.16: Let (M,τ) be a M-topological space. Then every closed M-set is a strongly b^*-closed M-set.

Proof: Assume that A is closed M-set in (M,τ) then $cl(A) = A$ with $C_{cl(A)} = C_A(x)$ for all $x \in X$ and U be any b-open M-set where $A \subseteq U$ with $C_A(x) \le C_U(x)$ for all $x \in X$.

Since $int(A) \le A$ with $C_{int(A)} \le C_A(x)$ for all $x \in X$, implies that $cl(int(A)) \subseteq U$ with $C_{cl(int(A))} \le C_U(x)$ for all $x \in X$. Hence A is sb^*-closed M-set in (M,τ).

Remark 3.17: The converse of the above theorem need not be true as seen from the following example.

Example 3.18: Let X= {a, b, c}, W = 1 and $M = \left\{\frac{1}{a},\frac{1}{b},\frac{1}{c}\right\}$, $\tau = \left\{\mathrm{M},\phi,\left\{\frac{1}{a}\right\},\left\{\frac{1}{a},\frac{1}{c}\right\}\right\}$ be a multiset topology on M.

Then (M,τ) be the M-topological space.

Now the Sb^* −closed M-sets of (M,τ) are M, ϕ, $\left\{\frac{1}{b}\right\},\left\{\frac{1}{c}\right\},\left\{\frac{1}{b},\frac{1}{c}\right\}$. In this M-topological space the sub M-set $A = \left\{\frac{1}{c}\right\}$ is sb^*- closed M-sets but not a closed M-set.

Theorem 3.19: Let (M,τ) and (N,σ) be any two M topological spaces. Let $f:(M,\tau)\to(N,\sigma)$ be a continuous M-set function then its sb^*-continuous M-set function but not conversely.

Proof: Let $f:(M,\tau)\to(N,\sigma)$ be a continuous M-set function. Let V be any open M-set in (N,σ). The inverse image of V is open M-set in (M,τ). Since every open M-set is sb^*- open M-set, inverse image of V is sb^*-open M-set in (M,τ). Therefore f is sb^*-continuous M-set function.

Remark 3.20: The converse of the above theorem need not be true as seen from the following example.

Example 3.21: Let X= {a, b, c}, W_1 = 1 and $Y = \{x,y\}$, $W_2 = 2$. Let $M = \left\{\frac{1}{a},\frac{1}{b},\frac{1}{c}\right\}$ and $N = \left\{\left\{\frac{2}{x}\right\},\left\{\frac{1}{y}\right\}\right\}$ be two M-sets.

Let $\tau = \left\{\mathrm{M},\phi,\left\{\frac{1}{a}\right\},\left\{\frac{1}{a},\frac{1}{c}\right\}\right\}$ and $\sigma = \left\{\mathrm{N},\phi,\left\{\frac{2}{x}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M,τ) and (N,σ) be two topological spaces. Now the sb^*-open M-set of (M,τ) are M, ϕ, $\left\{\frac{1}{a}\right\},\left\{\frac{1}{a},\frac{1}{c}\right\},\left\{\frac{1}{a},\frac{1}{b}\right\}$. Let the M-set function $f:(M,\tau)\to(N,\sigma)$ be defined as $f = \left\{\frac{\left\{\frac{1}{a},\frac{2}{x}\right\}}{2}, \frac{\left\{\frac{1}{b},\frac{2}{x}\right\}}{2}, \frac{\left\{\frac{1}{c},\frac{1}{y}\right\}}{1}\right\}$. Then f is sb^*- continuous M-set function. But f is not continuous M-set function

since for the open M-set $U = \left\{\frac{2}{x}\right\}$ in (N, σ), $f^{-1}(U) = \left\{\frac{1}{a}, \frac{1}{b}\right\}$ is not open M-set in (M, τ).

Remark 3.22: The following example shows that the g-continuous M-set function and sb^*-continuous function are independent.

Example 3.23: Let X= {x, y, z}, W_1 = 2 and $Y = \{a, b\}$, $W_2 = 1$.

Let $M = \left\{\frac{2}{x}, \frac{2}{y}, \frac{1}{z}\right\}$ and $N = \left\{\frac{1}{a}, \frac{1}{b}\right\}$ be two M-sets. Let $\tau = \left\{\text{M}, \phi, \left\{\frac{1}{x}\right\}, \left\{\frac{2}{x}\right\}\right\}$ and $\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{b}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M, τ) and (N, σ) be two M-topological spaces.

Let the M-set function $f: (M, \tau) \to (N, \sigma)$ be defined as $f = \left\{\frac{\left\{\frac{2}{x}, \frac{1}{a}\right\}}{2}, \frac{\left\{\frac{2}{y}, \frac{1}{a}\right\}}{2}, \frac{\left\{\frac{1}{z}, \frac{1}{b}\right\}}{1}\right\}$. Hence this M-set function f is g-continuous but not sb^*-continuous. Since the inverse image of open M-set $V = \left\{\frac{1}{b}\right\}$ in (N, σ) and $f^{-1}(V) = \left\{\frac{1}{z}\right\}$ is not sb^*- open M-set in (M, τ).

Example 3.24: Let X= {a, b, c}, W_1 = 1 and $Y = \{x, y\}$, $W_2 = 1$.

Let $M = \left\{\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right\}$ and $N = \left\{\left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}\right\}$ be two M-sets. Let $\tau = \left\{\text{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}\right\}$

and $\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{x}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M, τ) and (N, σ) be two topological spaces. Now the g-open M-set of (M, τ) are $\text{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{c}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}$ and sb^*open M-sets are $\text{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}$.

Let the M-set function $f: (M, \tau) \to (N, \sigma)$ be defined as $f = \left\{\frac{\left\{\frac{1}{a}, \frac{1}{x}\right\}}{1}, \frac{\left\{\frac{1}{b}, \frac{1}{x}\right\}}{1}, \frac{\left\{\frac{1}{c}, \frac{1}{y}\right\}}{1}\right\}$. Hence the inverse image of open M-set $\left\{\frac{1}{x}\right\}$ in (N, σ) is $\left\{\frac{1}{a}, \frac{1}{b}\right\}$ in (M, τ) which is sb^*- open M-set but not g-open M-set. Therefore this M-set function is sb^*-continuous but not g-continuous M-set function.

Remark 3.25: The following example shows that the αg-continuous M-set function and sb^*-continuous function are independent.

Example 3.26: Let X= {a, b, c}, W_1 = 1 and $Y = \{x, y\}$, $W_2 = 1$. Let $M = \left\{\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right\}$ and $N = \left\{\left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}\right\}$ be two M-sets.

Let $\tau = \left\{\text{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}\right\}$ and $\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{x}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M, τ) and (N, σ) be two M-topological spaces. Now the αg-open M-set of (M, τ) are $\text{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{c}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}$ and sb^*open M-sets are $\text{M}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a}, \frac{1}{b}\right\}, \left\{\frac{1}{a}, \frac{1}{c}\right\}$.

Let the M-set function $f: (M, \tau) \to (N, \sigma)$ be defined as $f = \left\{\frac{\left\{\frac{1}{a}, \frac{1}{x}\right\}}{1}, \frac{\left\{\frac{1}{b}, \frac{1}{x}\right\}}{1}, \frac{\left\{\frac{1}{c}, \frac{1}{y}\right\}}{1}\right\}$. This M-set function is sb^*-continuous but not αg-continous. Since the inverse image of open set $V = \left\{\frac{1}{x}\right\}$ in (N, σ) is $f^{-1}(V) = \left\{\frac{1}{a}, \frac{1}{b}\right\}$ in (M, τ) is not an αg-open M-set.

Example 3.27: Let X= {x, y, z}, W_1 = 2 and $Y = \{a, b, c\}, W_2 = 2$. Let $M = \left\{\frac{2}{x}, \frac{2}{y}, \frac{1}{z}\right\}$ and $N = \left\{\frac{2}{a}, \frac{1}{b}, \frac{1}{c}\right\}$ be two M-sets.

Let $\tau = \left\{\text{M}, \phi, \left\{\frac{1}{x}\right\}, \left\{\frac{2}{x}\right\}\right\}$ and $\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{c}\right\}\right\}$ be two M-topological space on M and N respectively.

Then (M, τ) and (N, σ) be two multiset topological spaces.

Let the M-set function $f: (M, \tau) \to (N, \sigma)$ be defined as $f = \left\{\frac{\left\{\frac{2}{x}, \frac{2}{a}\right\}}{4}, \frac{\left\{\frac{2}{y}, \frac{1}{b}\right\}}{2}, \frac{\left\{\frac{1}{z}, \frac{1}{c}\right\}}{1}\right\}$.

Here the inverse image of the open multiset

$V = \left\{\frac{1}{c}\right\}$ in (N, σ) is $\left\{\frac{1}{z}\right\}$ in (M, τ) is αg- open M-set but not sb^*- open M-set. Therefore the defined function is αg-continuous but not sb^*- continuous M-set function.

Remark 3.28: The following example shows that the sb^*-continuous M-set function and sg-continuous M-set function are independent.

Example 3.29: Let X= {x, y, z}, W_1 = 2 and $Y = \{a, b\}, W_2 = 1$. Let $M = \left\{\frac{2}{x}, \frac{2}{y}, \frac{1}{z}\right\}$ and $N = \left\{\frac{1}{a}, \frac{1}{b}\right\}$ be two M-sets.

Let $\tau = \left\{\text{M}, \phi, \left\{\frac{2}{x}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{2}{x}, \frac{2}{y}\right\}\right\}$ and

$\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{a}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M, τ) and (N, σ) be two M-topological spaces. Now the sg-open M-set of (M, τ) are

$\text{M}, \phi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{z}\right\}, \left\{\frac{2}{y}, \frac{1}{z}\right\}, \left\{\frac{1}{y}, \frac{1}{z}\right\}, \left\{\frac{2}{x}, \frac{1}{z}\right\},$ $\left\{\frac{1}{x}, \frac{1}{z}\right\}, \left\{\frac{2}{x}, \frac{1}{y}, \frac{1}{z}\right\}, \left\{\frac{1}{x}, \frac{2}{y}, \frac{1}{z}\right\}, \left\{\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right\}$

And sb^*- open M-set of

(M,τ) are M, ϕ, $\left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{2}{y}\right\}$, $\left\{\frac{1}{y}\right\}, \left\{\frac{1}{x},\frac{2}{y},\frac{1}{z}\right\}, \left\{\frac{2}{x},\frac{1}{y},\frac{1}{z}\right\}, \left\{\frac{1}{x},\frac{1}{y},\frac{1}{z}\right\}, \left\{\frac{2}{x},\frac{2}{y}\right\}, \left\{\frac{1}{x},\frac{2}{y}\right\}$,

$$\left\{\frac{2}{x},\frac{1}{y}\right\}, \left\{\frac{1}{x},\frac{1}{y}\right\}.$$

Let the M-set function f be defined as

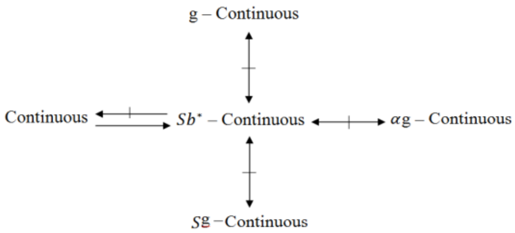
$$f = \left\{\frac{\left\{\frac{2}{x},\frac{1}{a}\right\}}{2}, \frac{\left\{\frac{2}{y},\frac{1}{a}\right\}}{2}, \frac{\left\{\frac{1}{z},\frac{1}{b}\right\}}{1}\right\}.$$

This M-set function f is sb^*-continuous but not sg-continuous. Since the inverse image of the open M-set $\left\{\frac{1}{a}\right\}$ in (N,σ) is $\left\{\frac{2}{x},\frac{2}{y}\right\}$ in (M,τ) is not sg-open M-set.

Example 3.30: Let X= {*x, y, z*}, $W_1 = 2$ and $Y = \{a,b\}$, $W_2 = 1$.

Let $M = \left\{\frac{2}{x},\frac{2}{y},\frac{1}{z}\right\}$ and $N = \left\{\frac{1}{a},\frac{1}{b}\right\}$ be two M-set. Let $\tau = \left\{\text{M}, \phi, \left\{\frac{2}{x}\right\}, \left\{\frac{2}{y}\right\} \left\{\frac{2}{x},\frac{2}{y}\right\}\right\}$ and $\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{b}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M,τ) and (N,σ) be two topological spaces. Let the M-set function $f:(M,\tau) \to (N,\sigma)$ be defined as $f = \left\{\frac{\left\{\frac{2}{x},\frac{1}{a}\right\}}{2}, \frac{\left\{\frac{2}{y},\frac{1}{a}\right\}}{2}, \frac{\left\{\frac{1}{z},\frac{1}{b}\right\}}{1}\right\}$. This M-set function f is sg-continuous but not sb^*-continous. Since the inverse image of open M-set $\left\{\frac{1}{b}\right\}$ in (N,σ) is $\left\{\frac{1}{z}\right\}$ in (M,τ) is not an sb^*- open M-set.

Remark 3.31: From the above results the diagramfollows:



4. STRONGLY b*- OPEN AND CLOSED M-SET FUNCTIONS

In this section we introduced the new concept of Sb^*- Open and Closed multiset function and studied some of their properties.

Definition 4.1: Let (M,τ) and (N,σ) be any two M topological spaces. Any M-set function $f:(M,\tau) \to (N,\sigma)$ is called strongly $b^* -$ closed ($sb^* -$closed) M-set function, if the image of every closed M-set in (M,τ) is $sb^* -$ closed M-set in (N,σ). The complement of a $sb^* -$ closed M-set is said to be $sb^* -$ open M-set.

Example 4.3: Let X= {x, y, z}, $W_1 = 1$ and $Y = \{a,b\}, W_2 = 2$.

Let $M = \left\{\frac{1}{x},\frac{1}{y},\frac{1}{z}\right\}$ and $N = \left\{\frac{2}{a},\frac{1}{b}\right\}$ be two M-sets. Let $\tau = \left\{\text{M}, \phi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{x},\frac{1}{z}\right\}\right\}$ and $\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a},\frac{1}{b}\right\}\right\}$ be two M-topological space on M and N respectively.

Then (M,τ) and (N,σ) be two M-topological spaces. Now the $sb^* -$ closed M-set of (N,σ) are N, ϕ, $\left\{\frac{2}{a}\right\}, \left\{\frac{1}{a},\frac{1}{b}\right\}, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{b}\right\}$ and the closed M-set of (M,τ) are M, ϕ, $\left\{\frac{1}{y},\frac{1}{z}\right\}, \left\{\frac{1}{y}\right\}$. Let the M-set function $f:(M,\tau) \to (N,\sigma)$ be defined as $f = \left\{\frac{\left\{\frac{1}{x},\frac{2}{a}\right\}}{2}, \frac{\left\{\frac{1}{y},\frac{1}{b}\right\}}{1}, \frac{\left\{\frac{1}{z},\frac{1}{b}\right\}}{1}\right\}$. This M-set function f is sb^*-closed.

Theorem 4.4: Let (M,τ) and (N,σ) be any two M-topological spaces. Then every closed M-set function is sb^*- closed but not conversely.

Proof: Let $f:(M,\tau) \to (N,\sigma)$ be closed M-set function and V be a closed M-set in (M,τ). Then $f(V)$ is closed M-set and hence $sb^* -$ closed M-set in (N,σ). Thus f is $sb^* -$ closed M-set function.

Remark 4.5: The converse of the above theorem need not be true as seen from the following example.

Example 4.6: Let X= {x, y, z}, $W_1 = 1$ and $Y = \{a,b,c\}, W_2 = 1$. Let $M = \left\{\frac{1}{x},\frac{1}{y},\frac{1}{z}\right\}$ and $N = \left\{\frac{1}{a},\frac{1}{b},\frac{1}{c}\right\}$ be two M-sets.

Let $\tau = \left\{\text{M}, \phi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{x},\frac{1}{y}\right\}\right\}$ and $\sigma = \left\{\text{N}, \phi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{a},\frac{1}{c}\right\}\right\}$ be two M-topological space on M and N respectively. Then (M,τ) and (N,σ) be two multiset topological spaces.

Now the $sb^* -$ closed M-set of (N,σ) are N, ϕ, $\left\{\frac{1}{b}\right\}, \left\{\frac{1}{c}\right\} \left\{\frac{1}{b},\frac{1}{c}\right\}$ and the closed M-set of (M,τ) are M, ϕ, $\left\{\frac{1}{y},\frac{1}{z}\right\}, \left\{\frac{1}{z}\right\}$.

Let the M-set function $f:(M,\tau) \to (N,\sigma)$ be defined as $f = \left\{\frac{\left\{\frac{1}{x},\frac{1}{a}\right\}}{1}, \frac{\left\{\frac{1}{y},\frac{1}{b}\right\}}{1}, \frac{\left\{\frac{1}{z},\frac{1}{c}\right\}}{1}\right\}$. This M-set function f is sb^*- closed but not closed as $f\left\{\frac{1}{z}\right\} = \left\{\frac{1}{c}\right\}$ is not closed M-set in (N,σ).

Theorem 4.7: Let (M,τ), (N,σ) and (P,η) be M-topological spaces. If a M-set function $f:(M,\tau) \to (N,\sigma)$ is closed and a M-set function $g:(N,\sigma) \to (P,\eta)$ is sb^*- closed then $g \circ f:(M,\tau) \to (P,\eta)$ is sb^*- closed M-set function.

Rajesh & Renuga

Proof: Let V be a closed M-set function in (M,τ). Since $f:(M,\tau)\to(N,\sigma)$ is closed. $f(V)$ is closed M-set in (N,σ). Since $g:(N,\sigma)\to(P,\eta)$ is sb^*-closed, $g(f(V))$ is sb^*- closed M-set of (P,η). Therefore $g\circ f:(M,\tau)\to(P,\eta)$ is sb^*- closed M-set function.

4. CONCLUSION

We have introduced the new concept of $\boldsymbol{Sb^*}$-Open, closed and continuous mutiset function in multiset topological space. Also studied some interesting properties of $\boldsymbol{Sb^*}$- open $\boldsymbol{sb^*}-$ closed and $\boldsymbol{sb^*}-$ continuous mutiset function in multiset topological space.

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